# The Sensitivity Analysis of Rainbow Options Based on Monte Carlo Simulation Method

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**Abstract:** A rainbow option is an option that allows the holder to decide whether it is a call or put before the expiration date or to choose the best of two alternative options which we focuses on in this article. The rainbow option to choose the best call option on underlying assets between NASDAQ and S&P 500 is under research in our study. First, to obtain the transmission that how the characteristics of NASDAQ and S&P 500 influence the option price, Monte Carlo Simulation method has been utilized to simulate the price path along with inputs including correlation between two underlying assets, risk-free-rate, historical return and market volatility. Based on that, sensitivity of option value has been conducted to figure out the relationship between the option value with all specific inputs. The result indicates that risk-free rate and volatility have a positive correlation with call value, and striking price has a negative one. In addition, we use the loglog plot and conclude that the volatility as opposed to risk-free-rate and striking price contributes the most to the fluctuations of option value. Finally, we extended our conclusion into the reality to predict the rainbow option price and analyze that to which extent the rainbow option would hedge risk compared with other common options. Given the situation that the financial market has been substantially influenced by Covid-19 and accompanied with a broad quantitative easing monetary policy, risk-free-rate is globally lower than before and financial market volatility climbs up higher. It’s reasonable to make a prediction that the risk-free-rate will stay low and strike price will remain stable for a long time, while the market volatility will decrease sharply by 30% to 60% due to the close-to-zero policy interest rate. Therefore, we conclude that the corresponding rainbow option price will drop significantly. In this case, rainbow option performs better for greatly hedging the volatility.

## 1. Introduction

Rainbow option, as a popular exotic option in the options market, has the right to let clients choose freely. The right to choose allows customers to have a higher degree of freedom and improve robustness, which helps hedging the risk. Rene M. Stulz proposed that choosing the smallest or largest option among the two risk assets will have a higher price, and this part of the premium comes from the ability to choose freely [1]. Herb Johnson stated a variety of risk assets General formula for calculating mixed options [2]. Carolalexander and Aanadvenkatramanan presents a recursive procedure for pricing European basket and rainbow options on N assets [3]. Hans N.E. Bystro¨m extended to a multivariate situation with covariance matrices of arbitrary size by simply using portfolios of simulated rainbow options [4]. Ye, L introduces an analytical method for multi-asset European options under a single-factor model [5]. This article aims to investigate the difference between rainbow option and other common stock options (such as call options of S&P 500 and NASDAQ) in terms of following aspects. First, how rainbow options perform differently with common options in terms of hedging risk based on the sensitivity analysis. Second, Whether the correlation between different underlying risk assets would influence the Rainbow option's performance on hedging risk. Finally, when facing a much more volatile financial market such as the one that is being shocked by Covid-19, whether rainbow options would greatly improve the ability of hedging risk.

To unfold the above questions, we simulate a series of option prices of a rainbow option with S&P 500 index and Nasdaq index being as underlying assets, based on which we conducted factor analysis, sensitivity analysis and further investigations. The data of S&P 500 index and Nasdaq index from March 2019 to March 2020 from yahoo-finance have been selected. A. Rasulov, R. Rakhmatov and A. Nafasov introduce an analytical method for multi-asset European options under a single-factor model [6]. With the input variables of historical return and correlation coefficient, we adopted Monte Carlo Simulation methods to simulate the option prices based on the assumption that stock price follows Brownian process ([6], [7]).

Here are some of our findings. First of all, we found that the same as common stock value options, the rainbow call option will also receive volatility, the positive effect of risk-free interest rates, and the negative effect of strike. Then we conducted a sensitivity analysis and found that volatility is the factor that has the greatest impact on stock options and rainbow options. Compared with common stock options, the price of rainbow will be further increased when facing more volatile markets.

In the second part, we study the impact of the correlation coefficient of two risk assets on the rainbow option. We find that the smaller the correlation coefficient, the higher the price of the rainbow option. At the same time, we found that when the low correlation coefficient and high volatility are combined, the price of the rainbow option will have a more dramatic increase, which means that the volatility and correlation coefficient have a certain degree of mutual promotion.

Finally, based on the above two conclusions, we observed the current market environment and predicted the possible trend of rainbow option in the future. We find that although market volatility has recently reached a record high, some of FED's policies, such as the Fed unlimited quantitative easing, can help ease market anxiety. Therefore, market volatility will decrease, and although the risk-free interest rate has a positive impact, it is difficult to see the effect of the risk-free rate in the current international background of zero interest rate. All in all, we predict that the price of rainbow options will fall in the future.

## 2. Methodology

To investigate the sensitivity of different inputs toward the price of rainbow option, this paper first simulates the price of two underlying assets through Monte Carlo Simulation methods and then carry out the sensitivity analysis based on the estimated option price.

### 2.1 Brownian Process of Stock Price

The stock price and index as well are broadly taken as consistent of Brownian Process. Suppose the price consistent of following stochastic process with R as shift rate and $\sigma^2$ as deviation rate:

$$ds = R(S,t)dt + \sigma(S,t)dz \tag{1}$$

Where the $R$ represents the expected return of stock price which is a time-dependent variable and a function of price itself. $\sigma(S,t)dz$ refers to the standard deviation of $ds$. Therefore the $ds$ is consistent of normal distribution with $R$ as mean and $\sigma^2$ as deviation. To further investigate the specific process of stock prices, we generated the deferential equation of $d\ln S$ based on Ito theorem as follows:

$$d\ln S = (R - 0.5\sigma^2)dt + \sigma dz \tag{2}$$

which indicates that the $\ln S$ is normally distributed with the mean being as $(R - 0.5\sigma^2)(T-t)$ and deviation of $\sigma^2$.

Therefore, denote the price at time T as $S_T$, then

$$\ln S_T \sim \phi\left[\ln S + (R - 0.5\sigma^2)(T-t), \sigma\sqrt{T-t}\right] \tag{3}$$

### 2.2 Monte Carlo Simulation of Option Prices

The derivatives are priced generally based on the uncertainties of their underlying assets. Monte Carlo Simulation methods are broadly applied to the calculation where there significantly needs measure the uncertainty. Therefore, given that the price of stock price is generally taken as consistent of stochastic process, the Monte Carlo Simulation would be the most favorable methods to simulate the stock price moving path.

Suppose that the return on the stock is normally distributed, but continuously compounded, leading to a lognormal distribution of the house's future value. If the expected return is $\alpha$, the risk-free rate is $r$, and the standard deviation of return is $\sigma$, the resulting distribution is lognormal with the following three properties:

1. $\ln(S_T)$ is normally distributed.

2. $E[\ln(S_T)] = \ln(S_0) + (\alpha - \frac{1}{2}\sigma^2)T$

3. $\mathrm{Stdev}[\ln(S_T)] = \sigma\sqrt{T}$

Using the relationship listed, with Z being a standard normally distributed random number, we arrive the formula of simulating the price of single stock (3):

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + Z\sigma\sqrt{T}} \tag{4}$$

Since there are two values of stocks needs to be simulated, and two values are correlative, we need to adjust the simulation formula slightly.

1. Calculate the correlation between two return of the stock price
2. Generate random number $Z_1$ (standard normally distributed)
3. Generate another random number Z
4. Calculate $Z_2$

$$Z_2 = Corr(R_1, R_2) \times Z_1 + \sqrt{1 - Corr^2(R_1, R_2)} \times Z \tag{5}$$

5. Adjusted formula:

$$S_{T_t} = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + Z_t\sigma\sqrt{T}},\ t = 1, 2 \tag{6}$$

Calculation of Option Values

1. Use the simulated stock price to calculate the return:

$$\text{return} = S_T/S_0 - 1 \tag{7}$$

2. According to the function:

Payoff (Yield)=max (max ($r_{S\&P}$ , $r_{NASDAQ}$) - X, 0)

3. Vice versa, the return of the put option:

Payoff (Yield)=max (X - min ($r_{S\&P}$ , $r_{NASDAQ}$), 0)

We calculated the payoff (yield of option) of the option through 1000 times of iteration in EXCEL, which will be used in the next procedure.

### 2.3 Estimation of Expected Return

#### 2.3.1 Linear Regression Model

We can find a linear relationship between payoff of the option and stock prices through the function of payoff. We can find the relation between $R_T$ and $S_T$ through the regression according to the equation of multiple linear regression:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \tag{8}$$

Replace the variable we need:

$$R_T = a + b_1 S_{T,1} + b_2 S_{T,2} \tag{9}$$

Then the present return is

$$R_0 = ae^{-rT} + b_1 S_{0,1} + b_2 S_{0,2} \tag{10}$$

### 2.3.2 Historical Return

Return at time T:

$$R_T = \frac{1}{n}\sum_{i=1}^{n} R_{T,i} \tag{11}$$

Then the present return:

$$R_0 = R_t e^{-rT} \tag{12}$$

### 2.4 Data Resources

Downloaded our necessary data, the stock price of S&P 500 and NASDAQ between March 21, 2019 and March 20, 2020.

**Volatility:** The VIX Index is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility.

Considering contemporary condition impacted by the COVID-19, we choose a average volatility which is nearly 0.25. But it is strongly influenced by the Fed's policy, which will fall from an exaggerated 200% to 300% to a stable 30% to 60%. So we predict it will keep in the level of 30% to 100% and give a range of answers.

**Risk-free rate:** Up to March 20, 2020, the risk-free rate is 0.0015, so we decided to confirm R as 0.0015.

**Alpha:** Equal to risk-free rate.

**Maturity:** We choose the maturity time as 1 year, 252 trade days.

**Strike point:** We choose a fiduciary value as 0.01.

## 3. The Simulation of Option Prices

### 3.1 The Expected Return of Two Assets

The empirical results of regression are presented in Table1.

Table 1. The regression of expected return.

| | I | II<br>No constant | III<br>Without Price1 | IV<br>Without Price2 |
|---|---|---|---|---|
| Constant | -0.53*** (-54.61) | | -0.53*** (-55.39) | -0.53*** (-54.85) |
| Price of S&P 500 | 0.00005*** (7.34) | 0.000058*** (1.42) | | 0.00023*** (56.03) |
| Price of NASDAQ | 0.00013*** (6.41) | 0.000013*** (0.12) | 0.000077*** (56.57) | |
| $R^2$ | 0.83 | 0.49 | 0.76 | 0.76 |
| Adj $R^2$ | 0.83 | 0.49 | 0.76 | 0.76 |

Note: ***represent the significance of 1%, **represents the significance of 5%, *represents the significance of 10%.

So the present return equation will be

$$R_0 = -0.5352784 \times e^{-0.0015 \times 1} + 0.0001291 \times S_{0,1} + 0.0000491 \times S_{0,2} \tag{13}$$

As shown in the table, both significance level of S&P and NASDAQ is lower than 0.01, so two dependent variable is prominent.

Substitute previous data we have got into the equation, we simulated the expected return as 0.106431.

### 3.2 Volatility of Underlying Assets

Since the volatility is uncertain, we assume that it may fluctuate in range from 30% to 60%, so we also did an interval simulation. Result is listed in the Table 2.

Table 2. Different payoff when volatility changes.

| Volatility | Payoff |
|---|---|
| 0.25 | 0.11 |
| 0.30 | 0.13 |
| 0.35 | 0.16 |
| 0.40 | 0.18 |
| 0.45 | 0.20 |
| 0.50 | 0.23 |
| 0.55 | 0.25 |
| 0.60 | 0.27 |
| 0.65 | 0.30 |
| 0.70 | 0.32 |
| 0.75 | 0.34 |
| 0.80 | 0.38 |
| 0.85 | 0.39 |
| 0.90 | 0.41 |
| 0.95 | 0.44 |
| 1.00 | 0.46 |

As shown in the Figure 1, the payoff of rainbow option goes up when volatility rises.

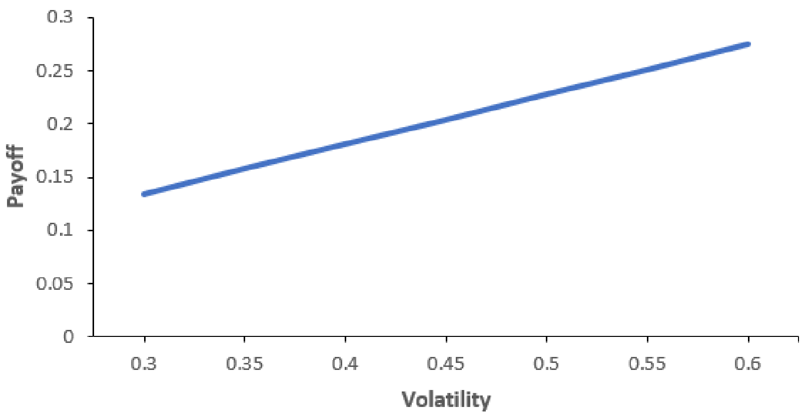


Figure 1. Different payoff when volatility changes.

### 3.3 The Simulation of Option Prices

The result of 1000 times simulation is listed in Figure 2 in the form of a line graph. Most of the prices are in the range of 0.2 to 0.8.

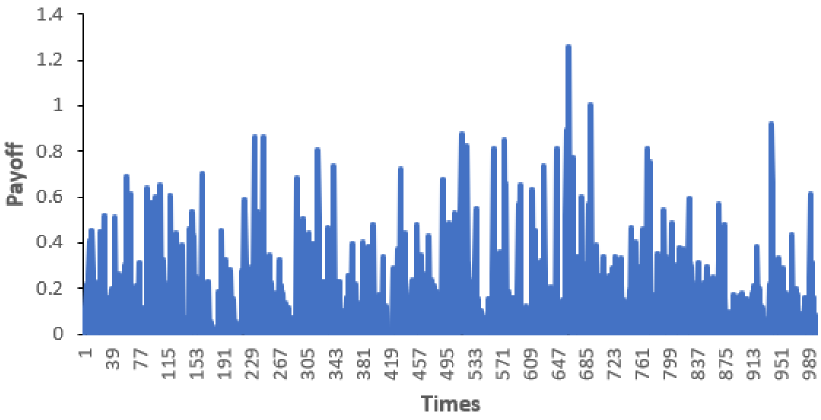


Figure 2. Simulation of option prices.

# 4. Results And Analysis

## 4.1 Trend Analysis

By changing different parameters, we can predict the future trend of this call-option price. Using the data table in the EXCEL, hold other parameters constant, change strike point and $\sigma$. Increase of the volatility will rise the value of the call, increase of the strike point fall the value of call. The relationship between the change of Strike point, Sigma, Risk-free rate and the value is shown in the Figure 3 below.

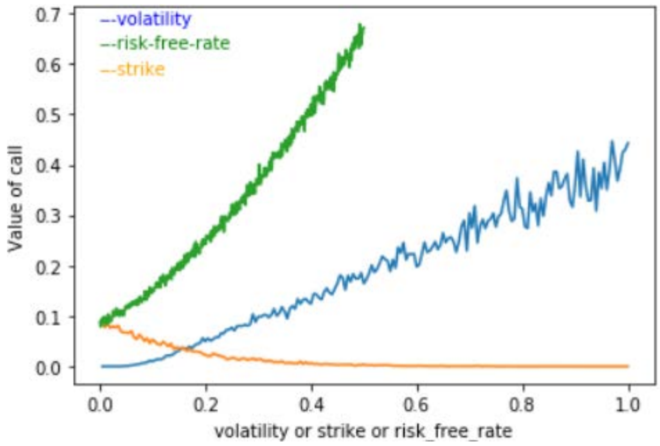


Figure 3. Plot of trend between the volatility, strike, risk-free rate and value of call option.

The following conclusions can be drawn from the chart:

Although there are some fluctuations in the curve, it is obviously that the increase of the volatility rises the value of call. Meanwhile, as volatility increases, the variance of call value is also greater, showing the first derivative of value call to volatility becomes infinity. Hence the impact of other variables will be relatively smaller. We can draw a conclusion that in more volatile financial markets (such as US stocks versus A shares), the value call is more unstable. The increase of the strike fall the value of call, and there is an exponential relationship between the two.

Moreover, there is increasing trend of the value when risk free-rate goes up, but the risk-free rate can hardly reach to 0.1 or higher than 0.1 in reality. The influence of risk free rate on call value gradually weakens with the increase of risk free rate, that is, when RF is at a lower level, the sensitivity of call value to risk free rate is higher. Below we will further analyze the sensitivity of call value to various variables.

A 3D plot below, Figure 4, showing the value of option is more sensitive to the change of the volatility.

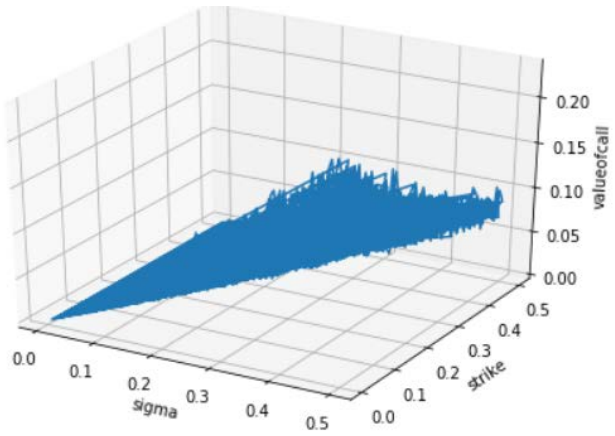


Figure 4. 3D graph of trend between the volatility, strike, risk-free rate and value of call option.

## 4.2 The Comparison Among Rainbow Options and Call Options

Because the rainbow option gives investor the opportunity to choose a higher yield between two different stock index options, it is reasonable that when variable changes, the return of the rainbow option will keep in a higher level compares to either two choices contained in rainbow option. We simulated the result in python through several iterations, and the result is roughly consistent with our hypothesis. As shown in Figure 5 and 6, blue curve which represents value of rainbow option overs other two.

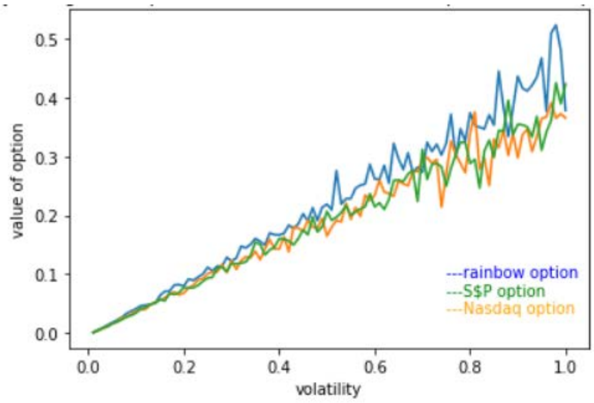


Figure 5. Value comparison when volatility changes.

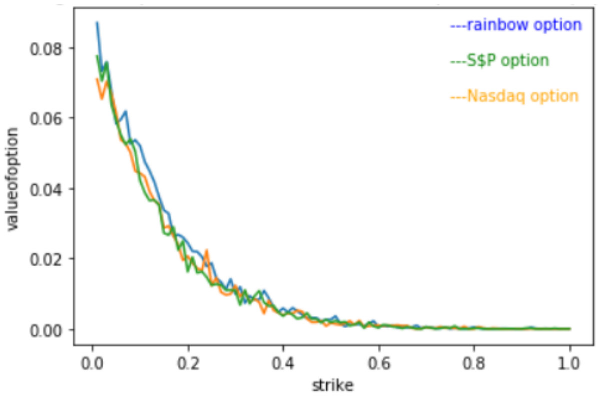


Figure 6. Value comparison when strike point changes.

Since S&P 500 and NADSAQ are both stock index, the correlation between two choices in rainbow is very high, which reaches the level of 0.9784. If we choose a rainbow option with a lower correlated alternatives, the advantage of rainbow option will be more obviously and distinctively. So we altered the correlation into 0.5. Results are shown in Figure 7 and 8. The value of rainbow option is significantly higher than the value of other two choices, and more sensitive to the strike than volatility.

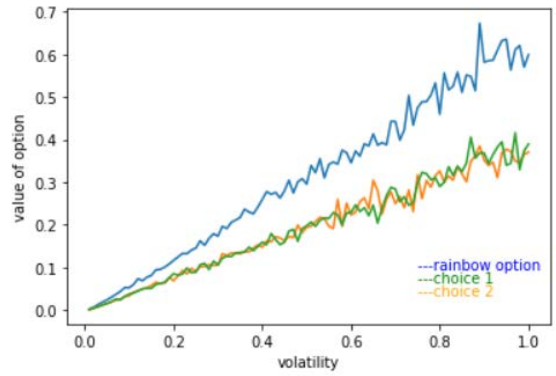


Figure 7. Value comparison when volatility changes.

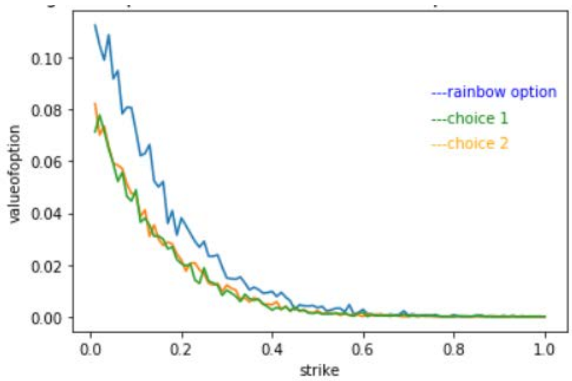


Figure 8. Value comparison when strike point changes.

Moreover, noticing that the correlation have obvious significance on the value of option, we also drew a value's trend graph about correlation in Figure 9. It is obvious in the graph that there is a negative relationship between and correlation, which clearly distinguished the rainbow option and two choices include in it.

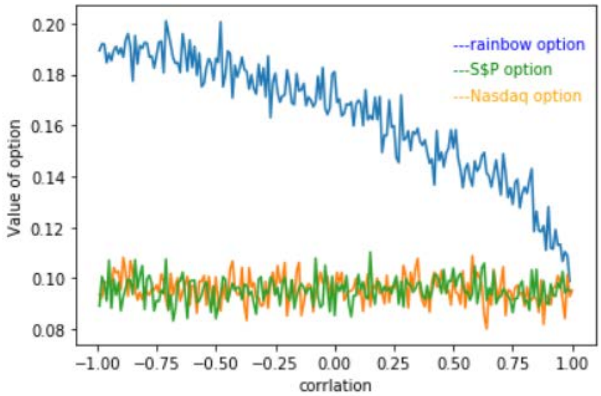


Figure 9. Relationship between correlation and value of the option.

If correlation is different, the impact of strike and volatility on rainbow option prices will also change. The lower the value of correlation, the more obvious the influence of volatility and strike on the value of the option. When correlation is negative, the impact will be more obvious.

As shown in Figure 10, although the value of rainbow option will fluctuate as the rise of volatility, rainbow option with negatively correlated choices are more valuable than which with positively correlated choices. Figure 11 indicate that in the range of 0.2 to 0.4, strike will influence more on rainbow option with negatively correlated choices, than option with positively correlated choices.

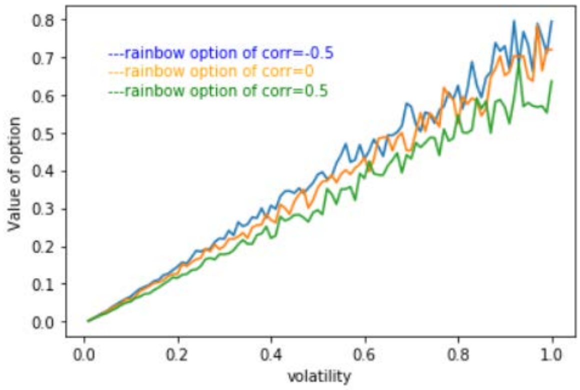


Figure 10. Influence different correlations have on changes in volatility.

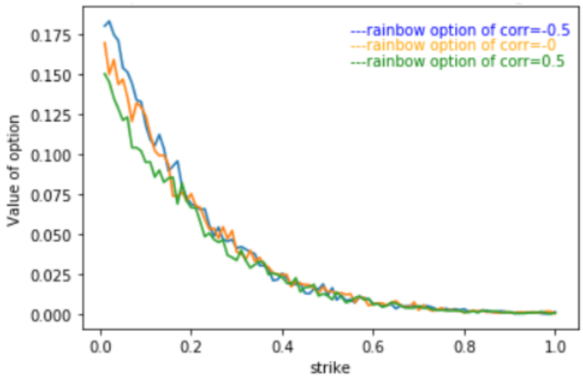


Figure 11. Influence different correlations have on changes in strike.

### 4.3 Sensitivity Analysis

We used loglog plot in Python to depict the fluctuation of the option value caused by the change of strike point, sigma, risk-free rate. Loglog plot is a function takes the logarithm to variables in both side of equation, which will clarify the fluctuation more clearly. The sensitivity of value to the change of three variables is shown in Figure 12, comparing to strike point and risk-free rate, volatility has more influence on value of the call.

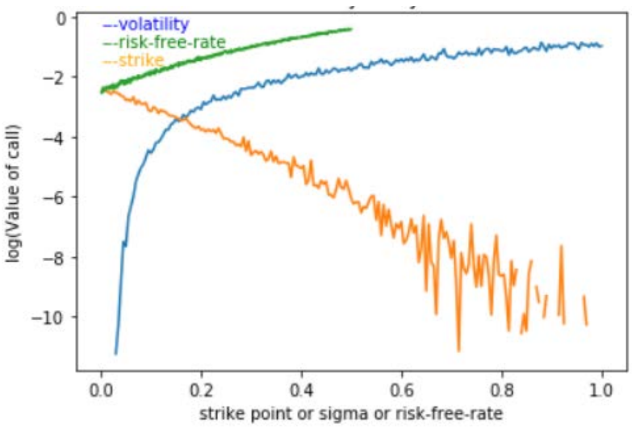


Figure 12. The relationship between the change of Risk-free Rate and the value (loglog plot)

The sensitivity of three different type of options is shown in the Figure 13, all of three options is sensitive to the change of strike points. But rainbow option is not very distinctive.

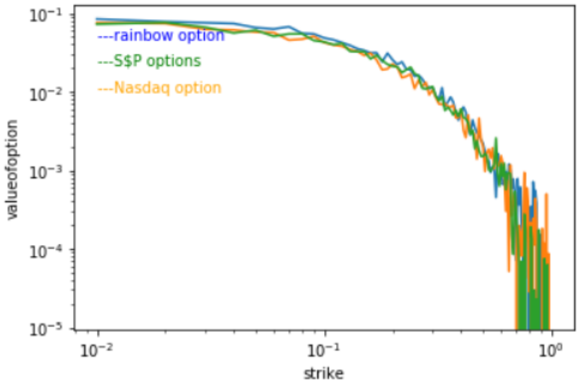


Figure 13. Comparison of S&P options, Nasdaq options and rainbow options

When we change the correlation into 0.5, the curve represents the value of rainbow option floats on the other two choices, which are shown in Figure 14.

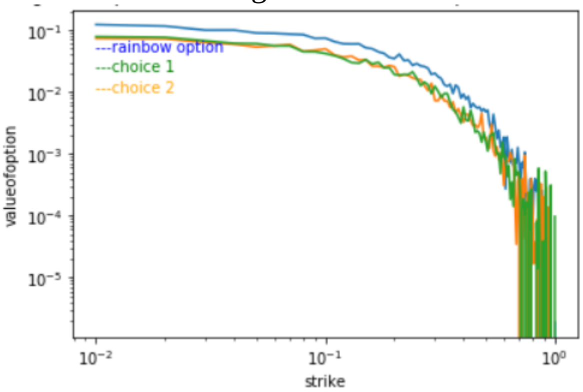


Figure 14. Comparison of rainbow option with two lower-correlated choices.

In the model, we assume that the Nasdaq and S&P 500 correlation and spots hold constant. We also hold other parameters constant, change risk-free rate, strike price and sigma, respectively.

We found when we keep risk free rate constant, other parameter had a small influence of the price. Although the risk-free rate can increase the value of this call, it hardly increases due to COVID-19. The reason for this is that the policy of the US government (FED), adopt Zero interest rate policy from March, which means zero interest rate. Moreover, FED also propose an unlimited Quantitative Easing Monetary Policy, and it is the first time FED support to buy enterprises bond and also provide loan to enterprises (see [8]). According to Dennis Lockhart [8], the president and CEO of the Federal Reserve Bank of Atlanta, this policy will have limits and also will increase the volatility of the market. Under this policy, the risk-free rate will also decrease, and then the bond trade is paying more than the risk-free rate, then everybody wants to buy it, and the price will increase. Although Sigma has already reaches to 200 these days and the volatility is already reached to it peak. However, we predict the volatility will become stable in the future. For example, according to BT [9], the global share market has the high volatility, which is significant volatility since February when the COVID-19 begins, and from the last few days it causes the crash in the global oil price. And, COVID-19 brings significant uncertainty to economic system, it brings an abrupt rise in financial volatility (see [10]). At the same time, Central Banks inject liquidity and cut interest rate by preventing credit crunch (see [10]). But, these trend will become stable in the future. As a result, the price of this call option will decrease.

## 5. Conclusion

Using NASDAQ and S&P 500 data, we analyze a rainbow option that customers can choose a better return. To be specific, we explored all variables that may affect option prices, and investigated to which extent these factors affect prices. Considering that the financial market are significantly much more volatile under COVID-19 than normal periods, we also try to predict the next option prices by interpreting existing planks. By changing of risk-free rate, we use Python and found there is an increasing trend of the risk-free rate, but the risk-free rate can hardly reach to 0.1 or higher than 0.1. In addition, we found that the increase of the volatility rises the value of call, and the increase of the strike x fall the value of call when we change of sigma and strike price. When we change strike price, we found the increase of the volatility can increase the value, but the increase of the striking point can fall the value. The trend is opposite relating to value. Based on Python, we can see the value of this option is more sensitive to the change of the volatility. Nowadays, because of COVID-19 situation, it has a very low risk-free rate, and very high implied volatility.

After we compare different factors that may influence sensitivity test, such as risk-free rate, volatility, striking point, we found that risk-free rate and volatility have a positive relationship with call, and strike has a negative relationship on call. And, when variable changes, the return of the rainbow option will keep in a higher level compares to either two choices contained in rainbow option. And, when using loglog plot, we also found that the price is significantly influenced by volatility compare to strike and risk-free rate.

The price of this rainbow option is higher than normal options, and it also has higher return. The higher the volatility, the higher the customers' bargaining power; the lower the correlation, the higher the customers' bargaining power. And, the lower the correlation, the bigger the variance and the more value of the option. Last, compare to normal option, our rainbow option give customers more option to choose, and has a high return. Moreover, it will be more expensive.

Our findings are consistent with a B-S model which has different factors that may influence the results. And, also according to FED zero interest policy based on current COVID-19 environment, the risk-free rate is decrease; the volatility is dramatically fall to 30%. Thus, we made a prediction that both the call and the put option price will decrease.

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